TUM Institute for Cognitive Systems (ICS)

OpenWalker

Module Description: Zero Moment Point (ZMPM)

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1 Module Description

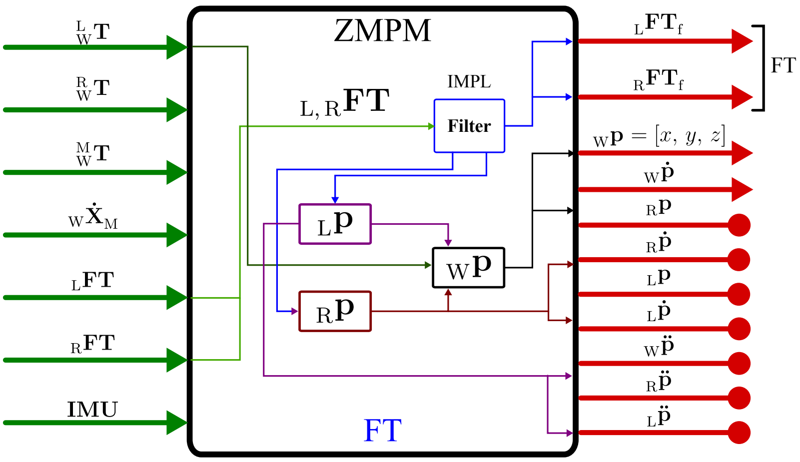

Figure 1.1: Zero Moment Point module: This module implements the local zmp estimation for each leg and the combined legs ${}_{W}\mathbf{p}$, ${}_{L}\mathbf{p}$, and ${}_{R}\mathbf{p}$, respectively.

The *Zero Moment Point* module (ZMPM) estimates the local Zero Moment Points (ZMP) for each foot and the global ZMP for both feet, ${}_{W}\mathbf{p}$, ${}_{L}\mathbf{p}$, and ${}_{R}\mathbf{p}$, respectively. The ZMP is an important concept for dynamics and control of legged locomotion, e.g., for humanoid robots. It specifies the point where the dynamic reaction forces at the contact of the foot with the ground does not produce any moment in the horizontal direction, i.e. the point where the total of horizontal inertia and gravity forces are in equilibrium. This module requires kinematic information of the feet, dynamic information of the Center of Mass (CoM), and ground reaction

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forces, which can be obtained with Force/Torque (FT) sensors, for example, mounted on the feet. The ZMP calculation can be extended using IMU sensors as well. The information obtained from the ZMP analysis is extremely important for balance, which is the highest priority task for legged robots. This module also filters the signals of the FT sensors, which are used by other components of the OpenWalker framework.

2 Module Connections

2.1 Inputs

Symbol	Name	Type	Description
${}^{L}_{W}\mathbf{T} \in \mathbb{R}^{4\times4}$	Left Foot Pose	`HomogeneousTransformation`	This matrix represents the pose of the left foot with respect to the world coordinate frame (wcf).
${}^{R}_{W}\mathbf{T} \in \mathbb{R}^{4\times4}$	Right Foot Pose	`HomogeneousTransformation`	This matrix represents the pose of the right foot with respect to the world coordinate frame (wcf).
${}^{M}_{W}\mathbf{T} \in \mathbb{R}^{4\times4}$	Real Robot CoM Pose	`HomogeneousTransformation`	This matrix represents the pose of the real CoM with respect to the world coordinate frame (wcf).
${}_{W}\dot{\mathbf{x}}_{M} \in \mathbb{R}^{6}$	Real Robot CoM Velocities	`CartesianVelocity`	This vector contains the real CoM velocities of the robot.
${}_{L}\mathbf{FT} \in \mathbb{R}^{6}$	Left Foot FT	`ForceTorqueSensor`	This vector contains the signals of the force/torque sensor mounted on the left foot.
${}_{R}\mathbf{FT} \in \mathbb{R}^{6}$	Right Foot FT	`ForceTorqueSensor`	This vector contains the signals of the force/torque sensor mounted on the right foot.
$\mathbf{IMU} \in \mathbb{R}^{10}$	IMU information	`ImuSensor`	This vector contains the IMU sensor information which includes Cartesian acceleration, angular position (in Quaternions), and angular velocity of the hip/torso.

2.2 Outputs

Symbol	Name	Type	Description
${}_{L}\mathbf{FT}_{f} \in \mathbb{R}^{6}$	Filtered Left Foot FT	`ForceTorqueSensor`	This vector contains the signals of the filtered force/torque sensor mounted on the left foot.
${}_{R}\mathbf{FT}_{f} \in \mathbb{R}^{6}$	Filtered Right Foot FT	`ForceTorqueSensor`	This vector contains the signals of the filtered force/torque sensor mounted on the right foot.
${}_{W}\mathbf{p} \in \mathbb{R}^{3}$	Zero Moment Point	`ZeroMomentPoint`	This vector represents the combined zero moment point with respect to the world coordinate frame.
${}_{W}\dot{\mathbf{p}} \in \mathbb{R}^{3}$	Zero Moment Velocity	`ZeroMomentPointP`	This vector represents the time derivative of the combined zero moment point with respect to the world coordinate frame.
${}_{W}\ddot{\mathbf{p}} \in \mathbb{R}^{3}$	Zero Moment Acceleration	`ZeroMomentPointPP`	This vector represents the 2nd time derivative of the combined zero moment point with respect to the world coordinate frame.
${}_{L}\mathbf{p} \in \mathbb{R}^{3}$	Left Foot Zero Moment Point	`ZeroMomentPoint`	This vector represents the zero moment point with respect to the left foot coordinate frame.
${}_{L}\dot{\mathbf{p}} \in \mathbb{R}^{3}$	Left Foot Zero Moment Velocity	`ZeroMomentPointP`	This vector represents the time derivative of the zero moment point with respect to the left foot coordinate frame.
${}_{L}\ddot{\mathbf{p}} \in \mathbb{R}^{3}$	Left Foot Zero Moment Acceleration	`ZeroMomentPointPP`	This vector represents the 2nd time derivative of the zero moment point with respect to the left foot coordinate frame.
${}_{R}\mathbf{p} \in \mathbb{R}^{3}$	Right Foot Zero Moment Point	`ZeroMomentPoint`	This vector represents the zero moment point with respect to the right foot coordinate frame.
${}_{R}\dot{\mathbf{p}} \in \mathbb{R}^{3}$	Right Foot Zero Moment Velocity	`ZeroMomentPointP`	This vector represents the time derivative of the zero moment point with respect to the right foot coordinate frame.
${}_{R}\ddot{\mathbf{p}} \in \mathbb{R}^{3}$	Right Foot Zero Moment Acceleration	`ZeroMomentPointPP`	This vector represents the 2nd time derivative of the zero moment point with respect to the right foot coordinate frame.

2.3 Inter-Connections

The inputs of the ZMPM come from different sources. The RRM provides the FT sensors information and the IMU sensor information, ${}_{L}\mathbf{FT}$, ${}_{R}\mathbf{FT}$, $\mathbf{IMU}$, respectively. The poses of the

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feet and the CoM with respect to the world coordinate frame $\left({}^{\mathrm{L}}_{\mathrm{W}}\mathbf{T}, {}^{\mathrm{R}}_{\mathrm{W}}\mathbf{T}, {}^{\mathrm{M}}_{\mathrm{W}}\mathbf{T}\right)$ as well as the CoM velocity $\left({}_{\mathrm{W}}\dot{\mathbf{x}}_{\mathrm{M}}\right)$ are provided by the FKM, using the real robot joint states $\left(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}\right)$.

This ZMPM provides the information of the combined zmp $\left({}_{\mathrm{W}}\mathbf{p}\right)$ and its time-derivative $\left({}_{\mathrm{W}}\dot{\mathbf{p}}\right)$ for the balancer (BM), and the filtered FT sensor signals $\left({}_{\mathrm{L}}\mathbf{FT}_{\mathrm{f}}, {}_{\mathrm{L}}\mathbf{FT}_{\mathrm{f}}\right)$ to compute the compliance of the feet in the FCM.

2.4 Common Methods

2.4.1 ZMP

The standard method to compute the ZMP is based directly on the FT sensor information of each foot [1].

First, the ZMP of each foot should be calculated:

$$
{}_{\mathrm{F}}\mathbf{p}_{\mathrm{x}} = \frac{-{}_{\mathrm{F}}\boldsymbol{\mu}_{\mathrm{y}} - {}_{\mathrm{F}}\mathbf{f}_{\mathrm{x}}\, d_{\mathrm{F}}}{{}_{\mathrm{F}}\mathbf{f}_{\mathrm{z}}} \tag{2.1}
$$

$$
{}_{\mathrm{F}}\mathbf{p}_{\mathrm{y}} = \frac{-{}_{\mathrm{F}}\boldsymbol{\mu}_{\mathrm{x}} - {}_{\mathrm{F}}\mathbf{f}_{\mathrm{y}}\, d_{\mathrm{F}}}{{}_{\mathrm{F}}\mathbf{f}_{\mathrm{z}}} \tag{2.2}
$$

$$
{}_{\mathrm{F}}\mathbf{p} = \left[{}_{\mathrm{F}}\mathbf{p}_{\mathrm{x}}, {}_{\mathrm{F}}\mathbf{p}_{\mathrm{y}}, 0\right]^{\top} \tag{2.3}
$$

where $\mathrm{F} = \{\mathrm{L}, \mathrm{R}\}$, ${}_{\mathrm{F}}\mathbf{FT} = \left[{}_{\mathrm{F}}\mathbf{f}, {}_{\mathrm{F}}\boldsymbol{\mu}\right]$, and d_{F} is the FT sensor offset of the foot F.

The ZMP of both feet is combined and represented with respect to the world coordinate frame. To this aim, first, we project each ZMP to the world coordinate frame (wcf) using the orientation of each feet w.r.t the wcf, ${}^{\mathrm{F}}_{\mathrm{W}}\mathbf{R} \in SO(3)$, which is obtained from the feet pose ${}^{\mathrm{F}}_{\mathrm{W}}\mathbf{T} \in \mathbb{R}^{4\times 4}$.

$$
{}_{\mathrm{W}}\mathbf{p}_{\mathrm{F}} = {}^{\mathrm{F}}_{\mathrm{W}}\mathbf{R}\, {}_{\mathrm{F}}\mathbf{p} \tag{2.4}
$$

Finally, each projected ZMP, $\left({}_{\mathrm{W}}\mathbf{p}_{\mathrm{F}}\right)$ with $\mathrm{F} = \{\mathrm{L}, \mathrm{R}\}$, is combined to produce the global ZMP w.r.t the wcf.

$$
{}_{\mathrm{W}}\mathbf{p}_{\mathrm{x}} = \frac{{}_{\mathrm{W}}\mathbf{p}_{\mathrm{R},x}\, {}_{\mathrm{W}}\mathbf{f}_{\mathrm{R},z} + {}_{\mathrm{W}}\mathbf{p}_{\mathrm{L},x}\, {}_{\mathrm{W}}\mathbf{f}_{\mathrm{L},z}}{{}_{\mathrm{W}}\mathbf{f}_{\mathrm{L},z} + {}_{\mathrm{W}}\mathbf{f}_{\mathrm{R},z}} \tag{2.5}
$$

$$
{}_{\mathrm{W}}\mathbf{p}_{\mathrm{y}} = \frac{{}_{\mathrm{W}}\mathbf{p}_{\mathrm{R},y}\, {}_{\mathrm{W}}\mathbf{f}_{\mathrm{R},z} + {}_{\mathrm{W}}\mathbf{p}_{\mathrm{L},y}\, {}_{\mathrm{W}}\mathbf{f}_{\mathrm{L},z}}{{}_{\mathrm{W}}\mathbf{f}_{\mathrm{L},z} + {}_{\mathrm{W}}\mathbf{f}_{\mathrm{R},z}} \tag{2.6}
$$

2.4.2 2nd Order Filtering

To filter the FT sensor signals, we can use a second order filter, for example, Butterworth filter [2]. This filter can be computed as:

$$
x_{\mathrm{f},k} = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} - a_1 x_{\mathrm{f},k-1} - a_2 x_{\mathrm{f},k-2} \tag{2.7}
$$

where a_i and b_j are the filter coefficients that depend on the cutoff frequency f_{c} as:

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$$b_0 = \frac{T^2\, w_c^2}{d} \tag{2.8}$$

$$b_1 = \frac{T^2\, 2\, w_c^2 0}{d} \tag{2.9}$$

$$b_2 = \frac{T^2\, w_c^2}{d} \tag{2.10}$$

$$a_0 = 1.0 \tag{2.11}$$

$$a_1 = \frac{T^2\, 2\, w_c^2 - 8}{d} \tag{2.12}$$

$$a_2 = \frac{T^2\, w_c^2 - 2\sqrt{2}\, T\, w_c + 4}{d} \tag{2.13}$$

References

[1] Englsberger, Johannes, et al. Three-dimensional bipedal walking control based on divergent component of motion, IEEE Transactions on Robotics, pp. 355-368, 2015.

[2] George Ellis. Filters in Control Systems, Chapter of Control System Design Guide (Fourth Edition), pp. 165-183, 2012.

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