TUM Institute for Cognitive Systems (ICS)

# OpenWalker

# Module Description: CoM Trajectory Generator (CoMTGM)

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February 14, 2020

## 1 Module Description

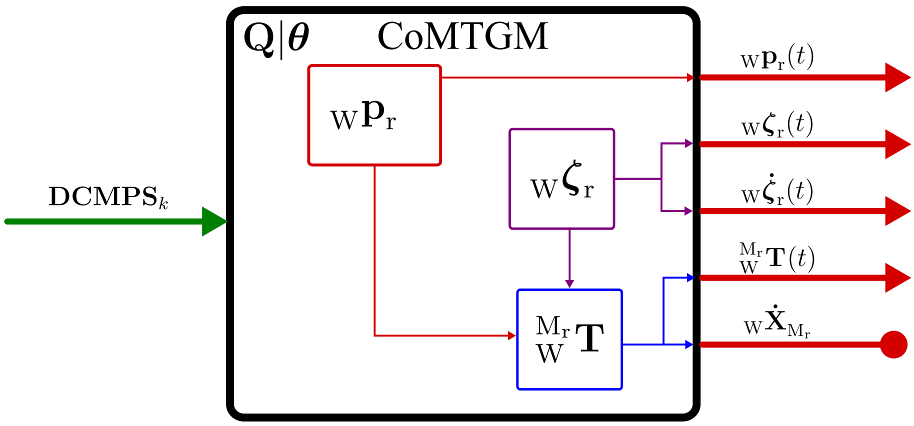


Figure 1.1: CoM Trajectory Generator module: This module generates a smooth reference trajectory for the center of mass by resolving the Divergent Component of Motion Dynamics.

This module receives the planned set of Divergent Component of Motion (DCM) way points and interpolates a smooth trajectory for both the DCM and the Center of Mass (CoM). The trajectories are generated following the DCM dynamics of the Linear Inverted Pendulum Model (LIPM). This module shifts the current way point to the next every time a step is finished, keeping the transition from one to the other continuous and smooth. The DCM moves

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 732287. 

between the Virtual Repellent Points (VRP) and the CoM follows it with the natural dynamics of the LIPM. While the CoM is a point in space, a common practice to consider whole-body angular momentum is to consider a virtual rigid body at the CoM with the same orientation of the base link (commonly the hip or the torso).

The computed trajectories are the reference Zero Moment Point with respect to the world ${}_{\mathrm{W}}\mathbf{p}_{\mathrm{r}} \in \mathbb{R}^3$, the reference DCM and its derivative with respect to the world ${}_{\mathrm{W}}\boldsymbol{\zeta}_{\mathrm{r}}$, ${}_{\mathrm{W}}\dot{\boldsymbol{\zeta}}_{\mathrm{r}} \in \mathbb{R}^3$, the reference CoM position and the orientation of the virtual link as an homogeneous transformation with respect to the world ${}^{\mathrm{M_r}}_{\mathrm{W}}\mathbf{T} \in \mathbb{R}^{4\times4}$, the velocity of the virtual link ${}_{\mathrm{W}}\dot{\mathbf{X}}_{\mathrm{M_r}} \in \mathbb{R}^6$.

# 2 Module Connections

## 2.1 Inputs

| Symbol | Name | Type | Description |
|---|---|---|---|
| $\{\mathbf{DCMPS}_i\}$ | DCP Point Set List | `DCMPointSetList` | A list containing the DCM way points needed to execute the planned footsteps. |

## 2.2 Outputs

| Symbol | Name | Type | Description |
|---|---|---|---|
| ${}_{\mathrm{W}}\mathbf{p}_{\mathrm{r}} \in \mathbb{R}^3$ | Reference ZMP | `ZeroMomentPoint` | Reference trajectory for the ZMP. |
| ${}_{\mathrm{W}}\boldsymbol{\zeta}_{\mathrm{r}} \in \mathbb{R}^3$ | Reference DCM Position | `DivergentComponentOfMotion` | Reference trajectory for the DCM. |
| ${}_{\mathrm{W}}\dot{\boldsymbol{\zeta}}_{\mathrm{r}} \in \mathbb{R}^3$ | Reference DCM Velocity | `DivergentComponentOfMotionP` | Derivative of the reference trajectory for the DCM. |
| ${}^{\mathrm{M_r}}_{\mathrm{W}}\mathbf{T} \in \mathbb{R}^{4\times4}$ | Reference CoM Position | `HomogeneousTransformation` | Reference trajectory for the CoM. This includes both position and orientation of a virtual link located at the CoM with the same oriantation of the base link. |
| ${}_{\mathrm{W}}\dot{\mathbf{X}}_{\mathrm{M_r}} \in \mathbb{R}^6$ | Reference CoM Velocity | `CartesianVelocity` | Derivative of the reference trajectory for the CoM. This includes both linear and angular velocities of the base link. |

## 2.3 Inter-Connections

This module receives the list of planned DCM waypoints from the DCM planner module. The outputs of this module is connected to the Balancer Module and the Command Generator Module. The balance module uses all the outputs while the Command Generator module uses only ${}^{\mathrm{M_r}}_{\mathrm{W}}\mathbf{T}$.

## 2.4 Common Methods

The dynamics of the LIPM are described by

$$\ddot{\mathbf{x}} = \omega^2 \left(\mathbf{x} - \mathbf{p}\right) \tag{2.1}$$

where $\mathbf{x} \in \mathbb{R}^3$ is the position of the CoM and $\ddot{\mathbf{x}} \in \mathbb{R}^3$ its second derivative, $\mathbf{p} \in \mathbb{R}^3$ is the position of the ZMP, and $\omega = \sqrt{\frac{g}{z}}$ is the pendulum parameter which define its natural frequency.

The DCM is the result of a change of variable of the LIPM equation which is defined as

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$$\boldsymbol{\zeta} = \mathbf{x} + \frac{\dot{\mathbf{x}}}{\omega} \tag{2.2}$$

$$\dot{\boldsymbol{\zeta}} = \dot{\mathbf{x}} + \frac{\ddot{\mathbf{x}}}{\omega} \tag{2.3}$$

with this representation, the equations 2.1 and 2.3 can be combined as

$$\dot{\mathbf{x}} = -\omega\left(\mathbf{x} - \boldsymbol{\zeta}\right) \tag{2.4}$$

$$\dot{\boldsymbol{\zeta}} = \omega\left(\boldsymbol{\zeta} - \mathbf{p}\right) \tag{2.5}$$

From (2.4) it is clear that the CoM follows the DCM in a first order stable dynamic system. However the dynamics of the DCM (2.5) are unstable. Nevertheless, these dynamics can be used to generate a stable reference trajectory for walking the $i$-th step by solving (2.5) for a constant $\mathbf{p} = \mathbf{r}_i$.

$$_{\mathrm{W}}\boldsymbol{\zeta}_{\mathrm{r}} = \mathbf{r}_i + \mathrm{e}^{\omega\left(t - t_{\mathrm{step}}\right)}\left(\boldsymbol{\zeta}_i - \mathbf{r}_i\right) \tag{2.6}$$

where $\mathbf{r}_i$ is the $i$-th Virtual Repellent Point (VRP) and $\boldsymbol{\zeta}_i$ is the $i$-th DCM point from the way points planned by the DCMP module.

Finally, (2.4) can be numerically solved using (2.6) as

$$_{\mathrm{W}}\mathbf{x}_{\mathrm{M_r},\,i+1} = {}_{\mathrm{W}}\boldsymbol{\zeta}_{\mathrm{r}} + \mathrm{e}^{-\omega t_{\Delta}}\left({}_{\mathrm{W}}\mathbf{x}_{\mathrm{M_r},\,i} - {}_{\mathrm{W}}\boldsymbol{\zeta}_{\mathrm{r}}\right) \tag{2.7}$$

where $t_{\Delta}$ is the iteration period.

## References

[1] Englsberger, Johannes, et al. "Bipedal walking control based on capture point dynamics." 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE, 2011.

[2] Englsberger, Johannes, et al. "Three-dimensional bipedal walking control based on divergent component of motion." Ieee transactions on robotics 31.2 (2015): 355-368.

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